INFORMATION THEORY

(4400:643)

Title: Information Theory Project – Capacity Analysis of One-Bit Quantized MIMO Systems with Transmitter Channel State Information Review

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Abstract

This report is a review of the paper "Capacity Analysis of One-Bit Quantized MIMO Systems with Transmitter Channel State Information" by Jianhua Mo and Robert W. Heath, Jr. The paper by Mo and Heath explores the capacity one-bit analog to digital converters in multiple input single out (MISO), single input multiple output (SIMO), and multiple input multiple output (MIMO) systems. The capacity of these systems are evaluated at infinite signal to noise ratios (SNR) as well as at a finite SNR.

Low resolution ADCs are applicable in high speed communication systems as they allow for faster data sampling at a lower power, relative to high resolution ADCs. By enabling the capability of faster sampling, the data rate may be increased thus allowing for faster communications. These low-resolution ADCs can be useful in millimeter-wave systems such as cellular 5G networks in which data rates can reach as high as 20Gbit/s as according to the IMT-2020 specifications.

This topic is relevant to information theory as it focuses on channel capacity and the effects on channel capacity as a result of the use of low-resolution ADCs at the receiver. Traditionally, receivers use relatively high-resolution ADCs to quantize the received signal. With single bit ADCs, this quantization becomes limited to 2 states. This proposed project will review the effects of such a system as discussed by Mo and Heath.

Intro

In a wireless communication channel, a transmitter communicates with a receiver. Various methods of modulation are available to communicate such as phase shift keying (PSK), amplitude shift keying (ASK), frequency shift keying (FSK), or a combination of these modulation techniques. Typically, each configuration consists of a single transmitter and a single receiver but the number of transmitters and or receivers can be increased to impact performance of the system. Figure 1 illustrates the various options for the system. Single input single output (SISO) consists of a single transmitter and single receiver. Single input multiple outputs (SIMO) is a system with one transmitter and multiple receivers. Multiple input single output (MISO) has multiple transmitters and a single receiver. And lastly, multiple input multiple output (MIMO) has multiple transmitters and receivers.

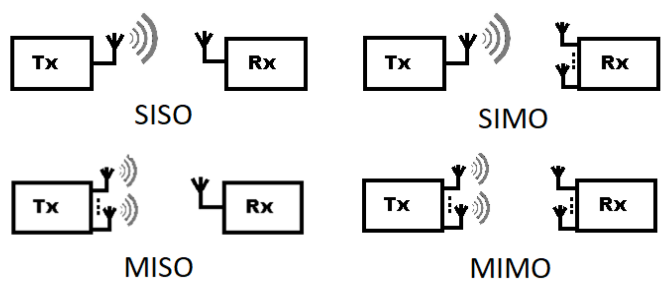

Figure 1: Transmitter - Receiver Configurations

Regardless of the modulation technique used, the received signal must be demodulated so that the original, unmodulated, signal can be extracted. Once the received signal is demodulated, the now analog signal must be converted to a digital signal. This is done with an analog to digital converter (ADC) and is typically done with a relatively high-resolution ADC. The use of high-resolution ADCs becomes an issue as bit-rate of the transmitted data increases. Figure 2 shows the relationship between the resolution of various types of ADCs and their respective sample rate. By observation of this plot, to achieve a higher sample rate, the resolution of the ADC must decrease.

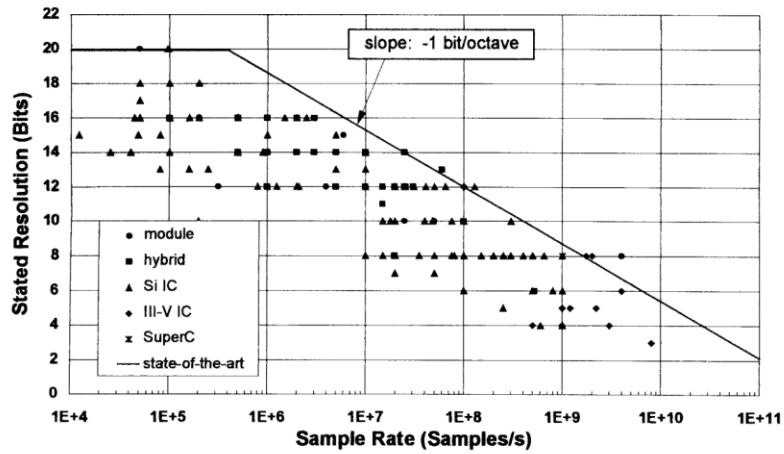

Figure 2: ADC Sample Rate vs Resolution

The second order effect of using lower resolution ADCs is the reduction in power consumption of the ADC. Figure 3 shows the structure of a 2-bit interpolation ADC. From this configuration, the extension to higher resolution ADCs will require exponentially more components and therefore require more power.

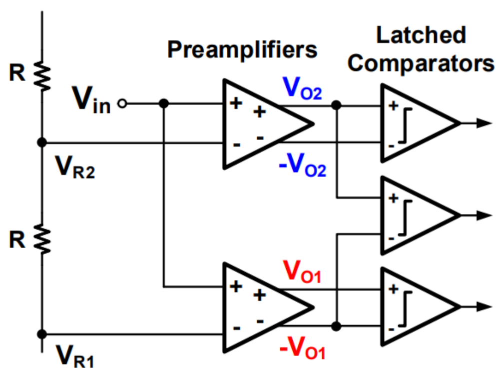

Figure 3: Simple ADC Structure

It is for these reasons that the use of low-resolution ADCs is appealing in a communication system. The use of low-resolution ADCs will sacrifice resolution of the quantization of the received signal. The effects of this binary quantization will be explored later.

By using one-bit ADCs, the use of some modulation techniques no longer become a valid choice. Such is the case for ASK and FSK. The number of symbols in ASK modulation is dependent on the resolution of the quantizer of the receiver, thus reducing the resolution of the receiver will have adverse effects on the performance of the system. Similarly, reducing the resolution of the receiver's quantizer in an FSK system will negatively impact performance. This means that the best performance will be achieved with PSK modulation as two-bit symbols can still be used as in the case of QPSK. Figure 4 illustrates such a system. Note that each receiver has two 1-bit ADCs. On ADC for the real part and one ADC for the imaginary part of the received signal. The number of transmitters is denoted as N_t and the number of receivers as N_r.

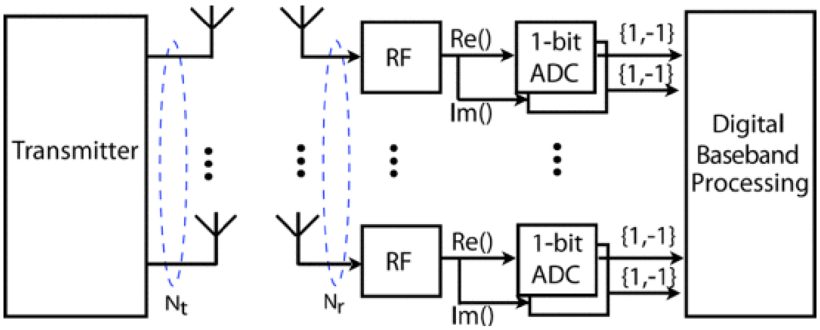

Figure 4: MIMO System with 1-bit Quantization

In the system of Figure 4, the received signal can be modeled as $\mathbf{y} = \mathbf{Hx} + \mathbf{n}$ where $\mathbf{H}$ is the channel matrix ($\mathbf{H}\epsilon\mathbb{C}^{N_r \times N_t}$), $\mathbf{x}$ is the signal sent by the transmitter ($\mathbf{x}\epsilon\mathbb{C}^{N_t \times 1}$), $\mathbf{y}$ is the received signal before quantization ($\mathbf{y}\epsilon\mathbb{C}^{N_r \times 1}$), and $\mathbf{n}$ is the circularly symmetric complex Gaussian noise ($\mathbf{n} \sim \mathcal{CN}(0, \mathbf{I})$). The real and imaginary part of the received signal have their own quantizer thus there are $2N_r$ one-bit resolution quantizers. The output after quantization is $\mathbf{r}=\mathrm{sgn}(\mathbf{y})=\mathrm{sgn}(\mathbf{Hx}+\mathbf{n})$ where $\mathbf{r}$ is the quantized output. Assuming channel state information at the transmitter (CSIT) and at the receiver (CSIR), the channel capacity with one-bit quantization is $\mathrm{C} = \max_{p(\mathbf{x}):\mathrm{tr}(\mathrm{E}(\mathbf{xx}^*))\leq \mathrm{P_t}} \mathrm{I}(\mathbf{x};\mathbf{r}\,|\mathbf{H})$ where mutual information, $\mathrm{I}(\cdot)$, is $\mathrm{I}(\mathbf{x}; \mathbf{r}|\mathbf{H}) = \int_{\mathbf{x}} \sum_{\mathbf{r}} \Pr(\mathbf{x})\Pr(\mathbf{r}|\mathbf{x}) \log_2 \frac{\Pr(\mathbf{r}|\mathbf{x})}{\Pr(\mathbf{x})}$ and P_t is the average power constraint at the transmitter. Finding a closed form solution to this mutual information expression is difficult, thus the capacity of a SIMO and MISO system is first evaluated as they form a simpler expression for capacity.

SISO Channel Capacity with One-Bit Quantization

Capacity of single input singe output (SISO) channel with one-bit quantization can be expressed as $C_{1\text{ bit, SISO}} = 2\left(1 - \mathcal{H}_b\left(Q(|h|\sqrt{P_t})\right)\right)$ where h is a scaler channel coefficient, $\mathcal{H}_b(p)$ is the binary entropy function such that $\mathcal{H}_b(p) = -p\log_2 p - (1-p)\log_2(1-p)$ and $Q(\cdot)$ is the tail probability of the standard normal distribution. Capacity is obtained by MRT beamforming and QPSK signaling in which $Pr\left[x = \sqrt{P_t}\frac{h}{\|h\|}e^{j\left(k\pi + \frac{\pi}{4}\right)}\right] = \frac{1}{4}, for\ k = 0,1,2, and\ 3$. The plot of $C_{1\text{ bit, SISO}}$ as a function of $|h|\sqrt{P_t}$ can be seen in Figure 5. From this plot, the upper bound of the capacity can be observed to be 2 bps/Hz.

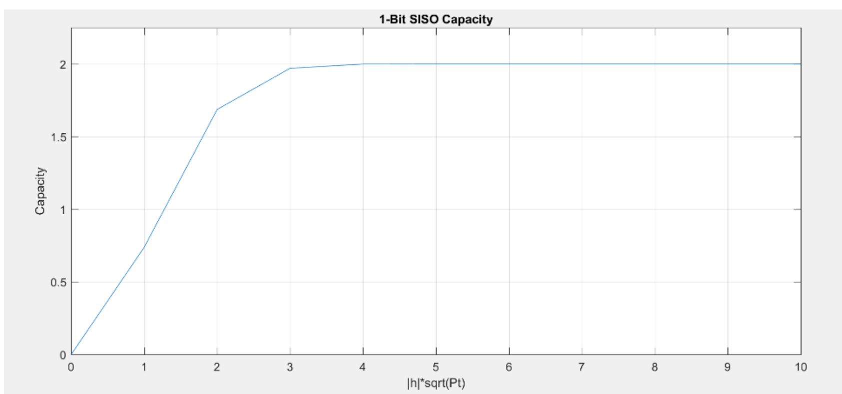

Figure 5: $C_{1\ bit,\ SISO}$ vs $|h|\sqrt{P_t}$

MISO Channel Capacity with One-Bit Quantization

In a multiple input single output system with 1-bit quantization the received signal is expressed as $r = sgn(y) = sgn(\boldsymbol{h} * \boldsymbol{x} + n)$ where **h** is the channel vector ($\mathbf{h} \epsilon \mathbb{C}^{N_t \times 1}$). In a typical MISO system in which quantization noise from the ADCs is negligible, or 0 in the case of infinite resolution ADCs, the optimal strategy is to use maximal ratio transmission (MRT) beamforming and Gaussian signaling. Capacity of MISO channel is 1-bit quantization can be express as $C_{1\text{ bit, MISO}} = 2\left(1 - \mathcal{H}_b\left(Q(\|\boldsymbol{h}\|\sqrt{P_t})\right)\right)$ where $\|\boldsymbol{h}\|$ is the norm of the channel vector $\boldsymbol{h}$. As in the case so SISO, capacity is obtained by maximal ratio transmission (MRT) beamforming and QPSK in which $Pr\left[\boldsymbol{x} = \sqrt{\boldsymbol{P_t}}\frac{\boldsymbol{h}}{||\boldsymbol{h}||}\boldsymbol{e}^{j\left(\boldsymbol{k\pi}+\frac{\pi}{4}\right)}\right] = \frac{1}{4}, for\ k = 0,1,2, and\ 3$. Again, the channel capacity has an upper bound of 2 bps/Hz. This can be observed by plotting channel capacity, $C_{1\text{ bit, MISO}}$, as a function of $||\boldsymbol{h}||\sqrt{P_t}$. The resulting plot will be the same as the $C_{1\text{ bit, SISO}}$ plot seen in Figure 5.

SIMO Channel Capacity with One-Bit Quantization

A SIMO channel with N_r antennas at the receiver has at most 2^{2N_t} possible quantization outputs. This is because each receiver has 2 ADCs, one for the real part and one for the imaginary part of the received signal. This means that the simple upper bound of channel capacity is $2N_r\ bps/Hz$. At infinite SNR, the capacity of a one-bit quantization SIMO channel, $\overline{C_{1-bit,SIMO}}$, is within the bounds $\log_2(4N_r) \leq \overline{C_{1-bit,SIMO}} \leq \log_2(4N_r + 1)$. The plot of this bound can be seen in Figure 6. Note that the upper bound of the capacity of such a channel is always an increasing function of the number of receivers.

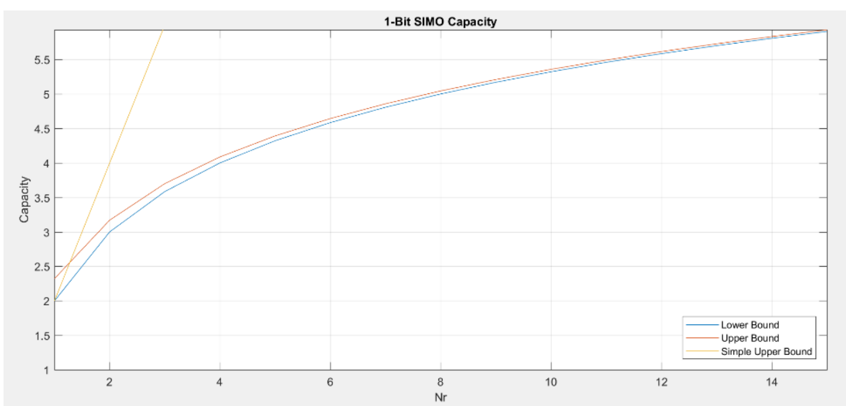

Figure 6: Bounded SIMO Capacity of Infinite SNR

MIMO Channel Capacity with One-Bit Quantization

In a 1-bit MIMO channel at infinite SNR, channel capacity, $\overline{C_{1-bit,MIMO}}$, is bounded within $\log_2(K(N_r, N_t)) \leq \overline{C_{1-bit,MIMO}} \leq \log_2(K(N_r, N_t) + 1)$ where $K(N_r, N_t) = 2 \cdot \sum_{k=0}^{2N_t-1} \binom{2N_r - 1}{k}$ when the number of transmitter is less than the number of receivers ($N_t < N_r$). When the number of transmitters is equal to or exceeds the number of receivers ($N_t \geq N_r$), the channel capacity is $\overline{C_{1-bit,MIMO}} = 2N_r$. The lower bound of this limit can be seen in the plot in Figure 7 and Figure 8.

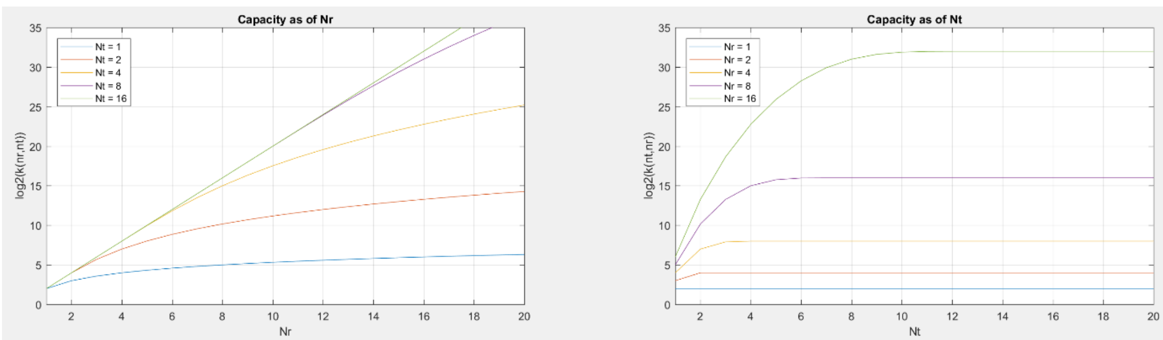

Figure 7: 1-bit MIMO Lower Bound Capacity at Infinite SNR N_r vs N_t Plot

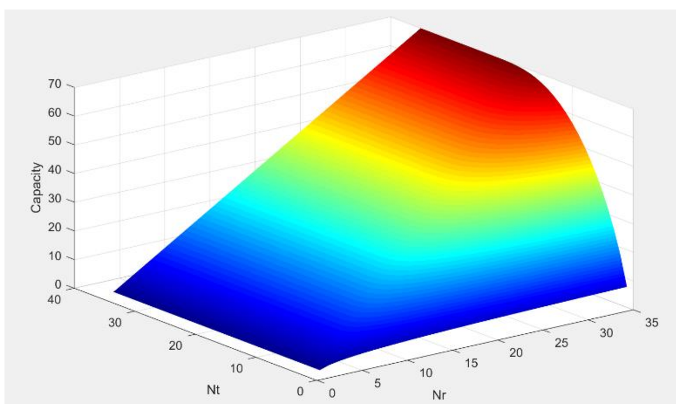

Figure 8: 1-bit MIMO Lower Bound Capacity at Infinite SNR Mesh

At a finite SNR MIMO channel capacity has an upper bound of $C_{1-bit}^{up} = 2N_r\left(1 - H_b\left(Q\left(\sqrt{\frac{P_t\sigma_{max}^2}{N_r}}\right)\right)\right)$ where σ_{max} is the largest singular value of **H.** From this equation, at high SNR the channel capacity approaches $2N_r$. This is because as P_t increases, the Q function approaches 0 and thus the binary entropy also approaches 0 resulting in $C_{1-bit}^{up} = 2N_r$. At low SNR, $C_{1-bit}^{up} = \frac{2}{\pi}\frac{P_t\sigma_{max}^2}{\ln 2} + o(P_t)$. The plot of the upper bound of a MIMO channel with finite SNR and 1-bit quantization can be seen in Figure 9 and Figure 10.

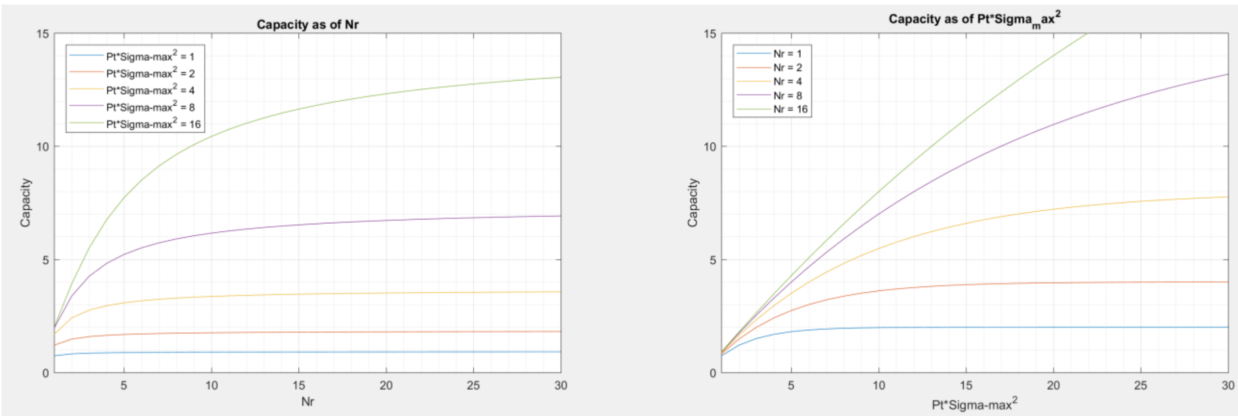

Figure 9: 1-bit MIMO Upper Bound Capacity at Finite SNR

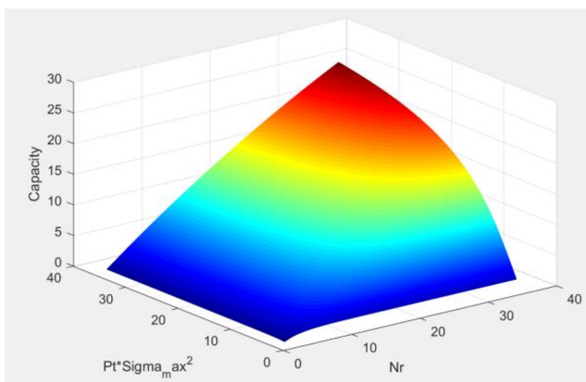

Figure 10: 1-bit MIMO Upper Bound Capacity at Finite SNR Mesh

Note that the unquantized MIMO channel capacity with channel state information CSIT is $C_{Unquantized}^{up} = \frac{P_t\sigma_{max}^2}{\ln 2} + o(P_t)$, therefore one-bit quantization results in at least power loss of

$Power\_Loss_{1-bit\ quantization} = 10 \cdot \log_{10}(C_{Unquantized}^{up}) - 10 \cdot \log_{10}(C_{1-bit}^{up}, low\_SNR) =$

$10 \cdot \log_{10}\left(\frac{2}{\pi}\right) \approx 1.96\ dB\ loss.$

References

MIMO Formats - SISO, SIMO, MISO, MU-MIMO. (n.d.). Retrieved from https://www.electronics-notes.com/articles/antennas-propagation/mimo/siso-simo-miso-mimo.php

Mo, J., & Heath, R. W. (2015). Capacity Analysis of One-Bit Quantized MIMO Systems With Transmitter Channel State Information. *IEEE Transactions on Signal Processing*,*63*(20), 5498-5512. doi:10.1109/tsp.2015.2455527

Walden, R. (1999). Analog-to-digital converter survey and analysis. *IEEE Journal on Selected Areas in Communications*,*17*(4), 539-550. doi:10.1109/49.761034

Zahrai, S., & Onabajo, M. (2018). Review of Analog-To-Digital Conversion Characteristics and Design Considerations for the Creation of Power-Efficient Hybrid Data Converters. *Journal of Low Power Electronics and Applications*,*8*(2), 12. doi:10.3390/jlpea8020012

Simulation Code

1-bit SISO Capacity

```
bound = 10;
i=1;

for k=0:1:bound
    C1bitsiso(i) = 2*(1-binary_entropy(qfunc(k)));
    i=i+1;
end

figure
Plot_1 = subplot(1,1,1);
x = 0:1:bound;
plot(x,C1bitsiso)
axis(Plot_1, [0 bound 0 2.25])
grid(Plot_1, 'on')
title(Plot_1,'1-Bit SISO Capacity')
ylabel(Plot_1,'Capacity')
xlabel(Plot_1,'|h|*sqrt(Pt)')


function y = binary_entropy(p)
y = -p*log2(p)-(1-p)*log2(1-p);
end
```

1-bit SIMO Capacity at Infinite SNR

```
Nr = 1:1:15;

for k=1:1:max(Nr)
Lower_Bound(k) = log2(4*Nr(k));
Upper_Bound(k) = log2(4*Nr(k)+1);
Simple_Upper(k) = 2*Nr(k);
end

figure
Plot_1 = subplot(1,1,1);
plot(Nr,Lower_Bound, Nr, Upper_Bound, Nr, Simple_Upper)
axis(Plot_1, [1 max(Nr) 1 max(Upper_Bound)])
grid(Plot_1, 'on')
title(Plot_1,'1-Bit SIMO Capacity')
ylabel(Plot_1,'Capacity')
xlabel(Plot_1,'Nr')
legend(Plot_1,{'Lower Bound','Upper Bound','Simple Upper

Bound'},'Location','southeast')
```

Infinite SNR Lower Bound Capacity of MIMO Channel

```
Nt = 1:1:35;
Nr = 1:1:35;
Capacity = zeros(max(Nt),max(Nr));
for i = 1:max(Nr)
    for j = 1:max(Nt)
        for k = 0:(2*j-1)
                if (2*i-1) < k
                    Capacity(i,j) = Capacity(i,j);
                else
                    Capacity(i,j) = Capacity(i,j) + nchoosek((2*i-1),k);
                    clc
                end
        end
        Capacity(i,j) = log2(2*Capacity(i,j));
    end
end
figure
Plot_0 = subplot(1,1,1);
colormap jet
[x y]= size(Capacity);
X = 1:1:x;
Y = 1:1:y;
surf(X,Y,Capacity)
shading interp
xlabel(Plot_0,'Nt')
ylabel(Plot_0,'Nr')
zlabel(Plot_0,'Capacity')
figure
Plot_1 = subplot(1,2,1);
x = 1:1:max(Nr);
plot(x,Capacity(:,1), x,Capacity(:,2), x,Capacity(:,4), x,Capacity(:,8),
x,Capacity(:,16))
axis(Plot_1, [1 20 0 35])
grid(Plot_1, 'on')
title(Plot_1,'Capacity as of Nr')
ylabel(Plot_1,'log2(k(nr,nt))')
xlabel(Plot_1,'Nr')
legend(Plot_1,{'Nt = 1','Nt = 2','Nt = 4','Nt = 8','Nt =
16'},'Location','northwest')

Plot_2 = subplot(1,2,2);
x = 1:1:max(Nt);
plot(x,Capacity(1,:), x,Capacity(2,:), x,Capacity(4,:), x,Capacity(8,:),
x,Capacity(16,:))
axis(Plot_2, [1 20 0 35])
grid(Plot_2, 'on')
title(Plot_2,'Capacity as of Nt')
ylabel(Plot_2,'log2(k(nt,nr))')
xlabel(Plot_2,'Nt')
legend(Plot_2,{'Nr = 1','Nr = 2','Nr = 4','Nr = 8','Nr =
16'},'Location','northwest')
```

Finite SNR Upper Bound Capacity of MIMO Channel

```
Nr = 1:1:35;
PtSigma = 1:1:35;
for j=1:max(Nr)
    for k=1:max(PtSigma)
        Capacity(j,k) = 2*Nr(j)*(1-
binary_entropy(qfunc(sqrt(PtSigma(k)/Nr(j)))));
    end
end
figure
Plot_0 = subplot(1,1,1);
colormap jet
surf(Nr,PtSigma,Capacity)
shading interp
xlabel(Plot_0,'Nr')
ylabel(Plot_0,'Pt*Sigma_max^2')
zlabel(Plot_0,'Capacity')
figure
Plot_1 = subplot(1,2,1);
plot(Nr,Capacity(:,1), Nr,Capacity(:,2), Nr,Capacity(:,4),
Nr,Capacity(:,8), Nr,Capacity(:,16))
axis(Plot_1, [1 30 0 15])
grid(Plot_1, 'on')
grid minor
title(Plot_1,'Capacity as of Nr')
ylabel(Plot_1,'Capacity')
xlabel(Plot_1,'Nr')
legend(Plot_1,{'Pt*Sigma-max^2 = 1','Pt*Sigma-max^2 = 2','Pt*Sigma-max^2
= 4','Pt*Sigma-max^2 = 8','Pt*Sigma-max^2 = 16'},'Location','northwest')
Plot_2 = subplot(1,2,2);
plot(PtSigma,Capacity(1,:), PtSigma,Capacity(2,:), PtSigma,Capacity(4,:),
PtSigma,Capacity(8,:), PtSigma,Capacity(16,:))
axis(Plot_2, [1 30 0 15])
grid(Plot_2, 'on')
grid minor
title(Plot_2,'Capacity as of Pt*Sigma_max^2')
ylabel(Plot_2,'Capacity')
xlabel(Plot_2,'Pt*Sigma-max^2')
legend(Plot_2,{'Nr = 1','Nr = 2','Nr = 4','Nr = 8','Nr =
16'},'Location','northwest')

function y = binary_entropy(p)
    y = -p*log2(p)-(1-p)*log2(1-p);
end
```

Additional Data

		Nt																																		
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
	2	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
	3	3.58	5.7	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
	4	4	7	7.91	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
	5	4.32	8.02	9.58	9.97	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
	6	4.58	8.86	11	11.8	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12
	7	4.81	9.56	12.2	13.5	13.9	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14
	8	5	10.2	13.3	15	15.8	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
	9	5.17	10.7	14.2	16.3	17.5	17.9	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18
	10	5.32	11.2	15	17.5	19	19.7	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20
	11	5.46	11.6	15.8	18.6	20.4	21.4	21.9	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22
	12	5.58	12	16.4	19.6	21.7	23	23.7	23.9	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24
	13	5.7	12.4	17.1	20.5	22.9	24.5	25.4	25.8	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26
	14	5.81	12.7	17.6	21.3	24	25.8	27	27.6	27.9	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28
	15	5.91	13	18.2	22.1	25	27.1	28.5	29.4	29.8	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30
	16	6	13.3	18.7	22.8	25.9	28.3	29.9	31	31.6	31.9	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32
	17	6.09	13.6	19.1	23.4	26.8	29.4	31.2	32.5	33.3	33.8	33.9	34	34	34	34	34	34	34	34	34	34	34	34	34	34	34	34	34	34	34	34	34	34	34	34
Nr	18	6.17	13.8	19.6	24.1	27.6	30.4	32.5	34	35	35.6	35.9	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36
	19	6.25	14	20	24.6	28.4	31.4	33.7	35.4	36.6	37.3	37.7	37.9	38	38	38	38	38	38	38	38	38	38	38	38	38	38	38	38	38	38	38	38	38	38	38
	20	6.32	14.3	20.3	25.2	29.1	32.3	34.8	36.7	38.1	39	39.6	39.8	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40
	21	6.39	14.5	20.7	25.7	29.8	33.1	35.8	37.9	39.5	40.6	41.3	41.7	41.9	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42
	22	6.46	14.7	21.1	26.2	30.5	34	36.8	39.1	40.8	42.1	43	43.5	43.8	44	44	44	44	44	44	44	44	44	44	44	44	44	44	44	44	44	44	44	44	44	44
	23	6.52	14.9	21.4	26.7	31.1	34.8	37.8	40.2	42.1	43.6	44.6	45.3	45.7	45.9	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46
	24	6.58	15.1	21.7	27.2	31.7	35.5	38.7	41.3	43.3	45	46.2	47	47.5	47.8	47.9	48	48	48	48	48	48	48	48	48	48	48	48	48	48	48	48	48	48	48	48
	25	6.64	15.3	22	27.6	32.3	36.2	39.5	42.3	44.5	46.3	47.6	48.6	49.3	49.7	49.9	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	26	6.7	15.4	22.3	28	32.8	36.9	40.3	43.2	45.6	47.6	49.1	50.2	51	51.5	51.8	51.9	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52
	27	6.75	15.6	22.6	28.4	33.4	37.6	41.1	44.2	46.7	48.8	50.4	51.7	52.6	53.3	53.7	53.9	54	54	54	54	54	54	54	54	54	54	54	54	54	54	54	54	54	54	54
	28	6.81	15.8	22.9	28.8	33.9	38.2	41.9	45.1	47.7	49.9	51.7	53.2	54.2	55	55.5	55.8	55.9	56	56	56	56	56	56	56	56	56	56	56	56	56	56	56	56	56	56
	29	6.86	15.9	23.1	29.2	34.4	38.8	42.6	45.9	48.7	51.1	53	54.6	55.8	56.7	57.3	57.7	57.9	58	58	58	58	58	58	58	58	58	58	58	58	58	58	58	58	58	58
	30	6.91	16.1	23.4	29.5	34.8	39.4	43.3	46.7	49.7	52.1	54.2	55.9	57.3	58.3	59	59.5	59.8	59.9	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60
	31	6.95	16.2	23.6	29.9	35.3	39.9	44	47.5	50.6	53.2	55.4	57.2	58.7	59.8	60.7	61.3	61.6	61.8	61.9	62	62	62	62	62	62	62	62	62	62	62	62	62	62	62	62
	32	7	16.3	23.9	30.2	35.7	40.5	44.7	48.3	51.5	54.2	56.5	58.5	60.1	61.3	62.3	63	63.5	63.8	63.9	64	64	64	64	64	64	64	64	64	64	64	64	64	64	64	64
	33	7.04	16.5	24.1	30.6	36.1	41	45.3	49	52.3	55.2	57.6	59.7	61.4	62.8	63.9	64.7	65.3	65.6	65.8	65.9	66	66	66	66	66	66	66	66	66	66	66	66	66	66	66
	34	7.09	16.6	24.3	30.9	36.5	41.5	45.9	49.8	53.1	56.1	58.7	60.8	62.7	64.2	65.4	66.3	67	67.5	67.7	67.9	68	68	68	68	68	68	68	68	68	68	68	68	68	68	68
	35	7.13	16.7	24.5	31.2	36.9	42	46.5	50.5	53.9	57	59.7	62	63.9	65.5	66.9	67.9	68.7	69.3	69.6	69.8	69.9	70	70	70	70	70	70	70	70	70	70	70	70	70	70

Figure 11: MIMSO Lower Bound Capacity Data Points

Orange – SISO, Blue – MOSI, Yellow – SIMO, Green - MIMO